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EFFECT OF SLIP CONDITION ON UNSTEADY MHD OSCILLATORY FLOW IN A CHANNEL FILLED WITH POROUS MEDIUM WITH HEAT RADIATION AND MASS TRANSFER

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ABSTRACT

The effect of slip condition on unsteady flow of an electrically conducting fluid trough a channel filled with saturated porous medium in the presence of transverse magnetic field and radiative heat transfer and mass transfer is studied. The dimensionless governing equations are solved using oscillatory flow conditions. The results are obtained for velocity, Sherwood Number and shearing stress for different parameters like Schmidt number, time, magnetic parameter, Darcy number, Renold number, Peclet number, rarefaction parameter etc. The flow characteristics are discussed and shown by means of graphs and tables.

KEYWORDS: Heat Transfer, Mass Transfer, MHD, Oscillatory Flow, Porous Medium and Slip Condition

INTRODUCTION

It is well known that at the microscopic level the fluid velocity matches the velocity of solid boundary. While in no slip boundary condition has been proven experimentally to be accurate for a number of microscopic flows, it is an assumption but based on physical principles. About two hundreds years ago Navier proposed a general boundary conditions for the fluid slip at solid boundary. That velocity of solid surface is proportional to shear stress at the surface[1] .i.e

$$V_x = h' \frac{\partial V_x}{\partial y}$$

where h' is the slip co-efficient. if $h' = 0$ then no slip condition obtain. if $h' \neq 0$ (finite)fluid slip occurs at the wall. The above relation is linear but one can establish a nonlinear relationship of the slip flow.

The fluid slippage phenomena of the solid boundaries appear in many applications such as micro channels or nanochannels and in application when surface is coated with special coatings such as thick monolayer of hydrophobic octadecyltrichlorosilance [9]. Recently ,several researchers have suggested that the no slip boundary condition may not suitable for hydrophilic flows over hydrophobic boundaries at both micro and nano scale[4,7-8].The effect of fluid slippage at the wall for coutte flow are considered by Marques et al.[6]under steady state condition and only for gases. The effect of slip condition on MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi[11]. Oscillatory free convection flow of a second order fluid from vertical plate have been studied by Mishra and Panda [3]. Soundalger et al. [5] have studied the effect of mass transfer in the flow past an infinite vertical oscillating plate in presence of constant heat flux .Makinde and Mhone[10] have studied the heat transfer on MHD oscillatory flow in a channel filled with porous medium. Senapati et.al.[12] have studied the effect of the mass and heat transfer effect on MHD oscillatory flow in a channel filled with porous medium.

In the recent paper we investigate the Effect of slip condition on unsteady MHD oscillatory flow in a channel filled with porous medium with heat radiation and mass transfer which is an extension work of Senapati et.al [12]

FORMULATION OF THE PROBLEM

The flow of a conducting optically thin fluid in a channel of width 'L' filled with saturated porous medium under the influence of an external applied homogenous magnetic field and radiative heat transfer is considered. It is assumed that Dufour and Soret effect viscous and joulean dissipation have been neglected. $X^{'}$-axis is taken along one wall of the channel and $Y^{'}$-axis is normal to it.

A uniform magnetic field of strength H_0 has been applied perpendicular to the wall of the channel and the magnetic permeability μ_e is constant throughout the field. $T_0^{'}$ and $T_w^{'}$ are the temperature and $C_0^{'}$ and $C_w^{'}$ are concentration at both walls of the channel. Then by usual Boussinesq's approximation the unsteady flow is governed by following equations:

$$\frac{\partial u^{'}}{\partial t^{'}} = -\frac{1}{\rho}\frac{\partial \mathrm{P}}{\partial x^{'}} + g\beta(T^{'} - T_{\infty}^{'}) + g\beta_C(C^{'} - C_{\infty}^{'}) + \nu\frac{\partial^2 u^{'}}{\partial y^{'2}} - \frac{\sigma B_0\ u^{'}}{\rho} - \frac{\nu}{\mathrm{K}}u^{'} \tag{1}$$

$$\rho C_p\frac{\partial T^{'}}{\partial t^{'}} = \kappa\frac{\partial^2 T^{'}}{\partial y^{'2}} - \frac{\partial q}{\partial y^{'}} \tag{2}$$

$$\frac{\partial C^{'}}{\partial t^{'}} = D\frac{\partial^2 C^{'}}{\partial y^{'2}} \tag{3}$$

Following Cogley *et al.* [2],it is assumed that the above plate is heated at a constant temperature so the radiative heat flux is given by

$$\frac{\partial q}{\partial y^{'}} = 4\alpha^2\left(T_0^{'} - T^{'}\right) \tag{4}$$

where $u^{'}$ is the axial velocity, P is the pressure, g is the gravitational force is radioactive heat flux, β is the co-efficient of expansion due to temperature, β_C is the co-efficient of expansion due to concentration, C_p is the specific heat at constant pressure, k is thermal conductivity, K is permeability co-efficient of porous medium,$B_0(\mu_e H_o)$ is the electromagnetic induction, σ is the conductivity of fluid, ρ is fluid density and ν is the kinematic viscosity co-efficient

The boundary conditions are as follows:

$$\left.\begin{aligned} & u^{'} = h^{'}\frac{\partial u^{'}}{\partial y^{'}}, T^{'} = T_w^{'}, C^{'} = C_w^{'}\ for y^{'} = L \\ & u^{'} = 0, T^{'} = T_0^{'}, C^{'} = C_0^{'}\ for y^{'} = 0 \end{aligned}\right\} \tag{5}$$

Let us introduce the dimensionless quantities as:

$$\left.\begin{aligned} & u=\frac{u'}{U}, t=\frac{t'U}{L}, y=\frac{y'}{L}, \theta=\frac{T'-T_0'}{T_w'-T_0'}, R_e=\frac{UL}{\nu}, G_r=\frac{g\beta\left(T_w'-T_0'\right)L}{U}, x=\frac{x'}{L} \\ & \phi=\frac{C'-C_0'}{C_w'-C_o'}, G_m=\frac{g\beta_c\left(C_w'-C_0'\right)L}{U}, \mathrm{P}_r=\frac{\mu c_P}{k}, S_c=\frac{\nu}{D}, \mathrm{M}=\mathrm{H}^2=\frac{\sigma B_0^2 L^2}{\rho\nu}, \\ & ,\mathrm{N}^2=\frac{4\alpha^2 L^2}{k}, \mathrm{P}_e=R_e.\mathrm{P}_r, D_a=\frac{\mathrm{K}}{L^2}, h=\frac{h'}{L} \end{aligned}\right\} \tag{6}$$

where D is mass diffusion, G_r is Grashof number , G_m modified Grashof number, H is Hartmann number , P_r is Prandtl number, S_c is Schmidt number, P_e is Peclet number , D_a is Darcy number , N is radiation parameter, U is flow mean velocity, h is rarefaction parameter and R_e is Renold number.

With the help of boundary conditions (5) and equation (4) the equations (1) to (3) reduce to

$$R_e\frac{\partial u}{\partial t}=-\frac{\partial \mathrm{P}}{\partial x}+\frac{\partial^2 u}{\partial Y^2}+G_r\theta+G_m\phi-\left(S^2+\mathrm{H}^2\right)u \tag{7}$$

$$\mathrm{P}_e\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial Y^2}+\mathrm{N}^2\theta \tag{8}$$

$$R_eS_c\frac{\partial\phi}{\partial t}=\frac{\partial^2\phi}{\partial Y^2} \tag{9}$$

Where $S^2=\dfrac{1}{D_a}$

The boundary conditions in non –dimensional form are

$$\left.\begin{aligned} & u=h\frac{\partial u}{\partial y}, \theta=1, \phi=1 \; for\; y=1 \\ & u=0, \theta=0, \phi=0 \; for\; y=0 \end{aligned}\right\} \tag{10}$$

METHOD OF SOLUTION

As the flow is purely oscillatory, in order to solve the equations (7) to (9) ,let us introduce

$$-\frac{\partial \mathrm{P}}{\partial x}=\lambda e^{i\omega t}, u(y,t)=u_0(y)e^{i\omega t}, \theta(y,t)=\theta_0(y)e^{i\omega t}, \phi(y,t)=\phi_0(y)e^{i\omega t} \tag{11}$$

where ω is the frequency of oscillation and λ is constant.

Then equations (7) to (9) becomes

$$\frac{d^2u_0}{dy^2}-m_1^2u_0+G_r\theta_0+G_m\phi_0+\lambda=0 \tag{12}$$

$$\frac{d^2\theta_0}{dy^2}+m_2^2\theta_0=0 \tag{13}$$

$$\frac{d^2\phi_0}{dy^2} - m_3{}^2\phi_0 = 0 \tag{14}$$

where $m_1 = \sqrt{S^2 + \mathrm{H}^2 + iR_e\omega}$, $m_2 = \sqrt{\mathrm{N}^2 - \mathrm{P}_e i\omega}$ and $m_3 = \sqrt{R_e S_c i\omega}$.

By solving the equations (12) to (14) we get

$$\phi_0 = \frac{\sinh(m_3 y)}{\sinh(m_3)} \tag{15}$$

$$\theta_0 = \frac{\sin(m_2 y)}{\sin(m_2)} \tag{16}$$

$$u_0 = \left(\begin{array}{l} \frac{\lambda}{m_1^2}\left(e^{-m_1}(1+hm_1)-1\right) + \frac{G_r}{m_1^2+m_2^2}(hm_2\cot m_2 - 1) \\ + \frac{G_m}{m_1^2-m_3^2}(hm_3\coth m_3 - 1) \end{array} \right) \frac{e^{m_1 y}}{2\sinh m_1 - 2hm_1\cosh m_1}$$

$$- \left(\begin{array}{l} \frac{\lambda}{m_1^2}\left(e^{m_1}(1-hm_1)-1\right) + \frac{G_r}{m_1^2+m_2^2}(hm_2\cot m_2 - 1) \\ + \frac{G_m}{m_1^2-m_3^2}(hm_3\coth m_3 - 1) \end{array} \right) \frac{e^{-m_1 y}}{2\sinh m_1 - 2hm_1\cosh m_1}$$

$$+ \frac{\lambda}{m_1^2} + \frac{G_r\sin(m_2 y)}{(m_1^2+m_2^2)\sin(m_2)} + \frac{G_m\sinh(m_3 y)}{(m_1^2-m_3^2)\sinh(m_3)}$$

$$= \left[\begin{array}{l} \left[\begin{array}{l} \left(-\frac{\lambda}{m_1^2}\right) + \frac{G_r}{m_1^2+m_2^2}(hm_2\cot m_2 - 1) \\ + \frac{G_m}{m_1^2-m_3^2}(hm_3\coth m_3 - 1) \end{array} \right] \left(\frac{\sinh m_1 y}{\sinh m_1 - hm_1\cosh m_1}\right) + \frac{hm_1\cosh m_1(y-1) + \sinh m_1(y-1)}{\sinh m_1 - hm_1\cosh m_1}\left(\frac{\lambda}{m_1^2}\right) \\ + \frac{\lambda}{m_1^2} + \frac{G_r\sin(m_2 y)}{(m_1^2+m_2^2)\sin(m_2)} + \frac{G_m\sinh(m_3 y)}{(m_1^2-m_3^2)\sinh(m_3)} \end{array} \right] \tag{17}$$

Again $\phi = \frac{\sinh(m_3 y)}{\sinh(m_3)} e^{i\omega t}$ (18)

$$\theta = \frac{\sin(m_2 y)}{\sin(m_2)} e^{i\omega t} \tag{19}$$

$$u = u_0 e^{i\omega t} \tag{20}$$

The dimensionless shearing stress at the upper wall of the channel is

$$\tau_1 = \left[\begin{array}{l} \left[\begin{array}{l} \left(-\dfrac{\lambda}{m_1^2}\right) + \dfrac{G_r}{m_1^2+m_2^2}\left(hm_2 \cot m_2 - 1\right) \\ + \dfrac{G_m}{m_1^2-m_3^2}\left(hm_3 \coth m_3 - 1\right) \end{array}\right]\left(\dfrac{m_1 \cosh m_1}{\sinh m_1 - hm_1 \cosh m_1}\right) + \dfrac{\lambda}{\left(\sinh m_1 - hm_1 \cosh m_1\right)m_1} \\ + \dfrac{G_r m_2 \cot(m_2)}{\left(m_1^2+m_2^2\right)} + \dfrac{G_m m_3 \coth(m_3)}{\left(m_1^2-m_3^2\right)} \end{array}\right] e^{i\omega t} \tag{21}$$

The dimensionless shearing stress at lower wall of the channel is

$$\tau_2 = \left[\begin{array}{l} \left[\begin{array}{l} \left(-\dfrac{\lambda}{m_1^2}\right) + \dfrac{G_r}{m_1^2+m_2^2}\left(hm_2 \cot m_2 - 1\right) \\ + \dfrac{G_m}{m_1^2-m_3^2}\left(hm_3 \coth m_3 - 1\right) \end{array}\right]\left(\dfrac{m_1}{\sinh m_1 - hm_1 \cosh m_1}\right) + \dfrac{\lambda\left(\cosh m_1 - hm_1 \sinh m_1\right)}{\left(\sinh m_1 - hm_1 \cosh m_1\right)m_1} \\ + \dfrac{G_r m_2}{\left(m_1^2+m_2^2\right)\sin(m_2)} + \dfrac{G_m m_3}{\left(m_1^2-m_3^2\right)\sinh(m_3)} \end{array}\right] e^{i\omega t} \tag{22}$$

Rate of heat transfer across the channels at the upper wall is

$$Nu_1 = -\left(\frac{\partial\theta}{\partial y}\right)_{y=1} = -\frac{m_2 \cos m_2}{\sin m_2} e^{i\omega t} \tag{23}$$

Rate of heat transfer across the channels at the lower wall is

$$Nu_2 = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\frac{m_2}{\sin m_2} e^{i\omega t} \tag{24}$$

The rate of mass transfer at the upper wall is

$$Sh_1 = -\left(\frac{\partial\phi}{\partial y}\right)_{y=1} = -\frac{m_3 \cosh m_3}{\sinh m_3} e^{i\omega t} \tag{25}$$

The rate of mass transfer at lower wall is

$$Sh_2 = -\left(\frac{\partial\phi}{\partial y}\right)_{y=0} = -\frac{m_3}{\sinh m_3} e^{i\omega t} \tag{26}$$

DISCUSSIONS AND GRAPHS

In this paper we have studied the effect of slip condition on unsteady MHD oscillatory flow in a channel filled with porous medium with heat radiation and mass transfer .The effect of the parameters Gr,h, Gm, H, N,w, D_a, P_r and S_c on flow characteristics have been studied and shown by means of graphs and tables. To obtain the graphs and tables the modulii of velocity shearing stress taken w.r.t distance(y). Effect of different parameterson shearing stress at both the plates are shown on the same graphs.

Figure-(1) illustrates the effect of the parameters Gr,Gm and H on velocity at any point of the fluid, when λ=1,Sc=0.22,S=1,h=1,w=1,t=1,Re=0.71,Pe=1 and N=2. It is noticed that the velocity increases with the increase of Grash of number (Gr) and modified Grash of number(Gm) from the lower wall to the upper wall. Further the velocity at any point of the fluid decreases as the increase of Hartmann number (H) from the lower wall to the upper wall.

Figure-(2) illustrates the effect of the parameters λ,Re and Pe on velocity at any point of the fluid, when H=1,Sc=0.22,S=1,h=1,w=1,t=1,Gr=1,Gm=1 and N=2.It is noticed that the velocity decreases from the lower wall to the upper wall with the increase of both Renold number (Re) and peclet number (Pe). Further the velocity at any point of the fluid increases as the increase of the parameter (λ) from the lower wall to the upper wall.

Figure-(3) illustrates the effect of the parameters N,S and h on velocity at any point of the fluid, when w=1,Re=0.71,Gr=1,t=1,Gm=1,Pe=1,H=1, λ =1 and Sc=0.22.It is noticed that the velocity at any point of the fluid increases with the increasing value of radiation parameter (N) from lower wall to upper wall. Further the velocity at any point of the fluid decreases with the increase of S and rarefaction parameter (h) from lower wall to upper wall.

Figure-(4) illustrates the effect of the parameters w,t and Sc on velocity at any point of the fluid, when H=1,Re=0.71,S=1,h=1,w=1, λ =1,Gr=1,Pe=1 .Gm=1 and N=2.It is noticed that the velocity decreases with the increase of frequency of oscillation(w) and Schmidt number(Sc) from the lower wall to upper wall. Further the velocity at any point of the fluid remain same with the increase of time (t) from the lower wall to the upper wall.

Figure-(5) illustrates the effect of the parameters Gr, Gm and H on shearing stress at both upper and lower wall, when t=1, Re=0.71, Pe=1, Sc=0.22 h=1,w=1 , S=1,λ =1 and N=2.It is noticed that the shearing stress increases with the increase of Grashof number(Gr) and modified Grashof number(Gm), also decreases with the increase of Hartmann number (H) at both walls .The shearing stress at upper wall is less than lower wall.

Figure-(6) illustrates the effect of the parameters Re, λ and Pe on shearing stress at both upper and lower wall, when t=1, Gm=1, Gr=1,Sc=0.22 h=1,w=1, S=1H =1 and N=2.It is noticed that the shearing stress increases with the increase of parameter (λ), also decreases with the increase of Renold number (Re) and peclet number (Pe) at both walls. The shearing stress at upper wall is less than lower wall.

Figure-(7), illustrates the effect of the parameters S, h and N on shearing stress at both upper and lower wall, when t=1, Gm=1, Gr=1, Re=0.71 Sc=0.22, Pe=1, H =1 and w=1.It is noticed that the shearing stress decreases with the increase with of increasing S and rarefaction parameter (h), where as increases with the increase of radiation parameter (N) at both the walls. The shearing stress at upper wall is less than lower wall.

Figure-(8) illustrates the effect of the parameters Sc and w on shearing stress at both upper and lower wall, when t=1, Gm=1, Gr=1, Re=0.71 h=1, Pe=1, S=1, H =1 and N=2. It is noticed that the shearing stress decreases with the increase both Schmidt number(Sc) and frequency of oscillation(w). The shearing stress at upper wall is less than lower wall.

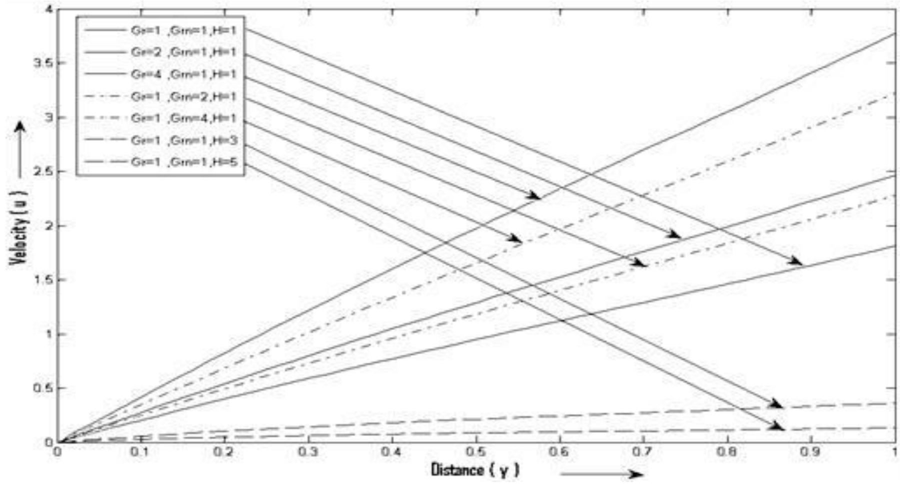

Figure 1: Effect of Gr, Gm and H on Velocity Profile when λ=1, Sc=0.22, S=1, h=1, w=1, t=1, Re=0.71, Pe=1 and N=2

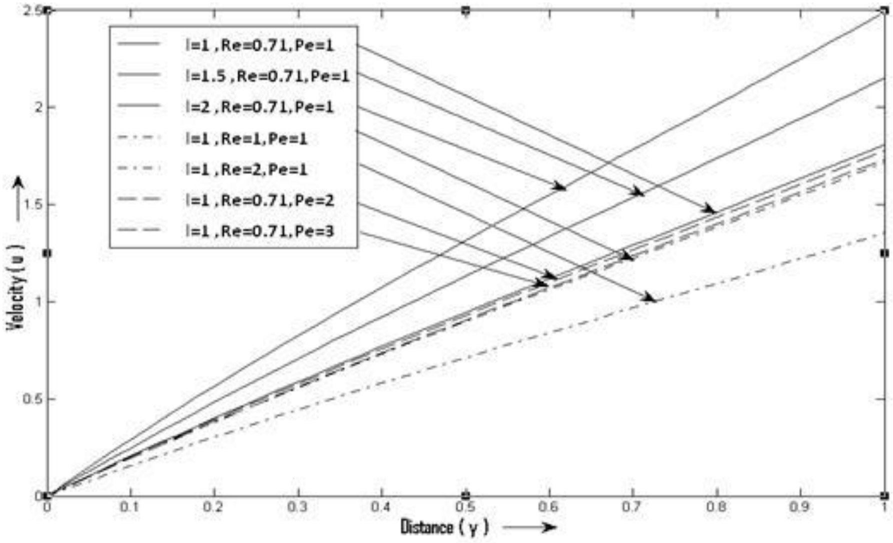

Figure 2: Effect of Re, Pe and $^{\lambda,(l)}$ on Velocity Profile when H=1, Sc=0.22, S=1, h=1, w=1, t=1, Gr=1, Gm=1 and N=2

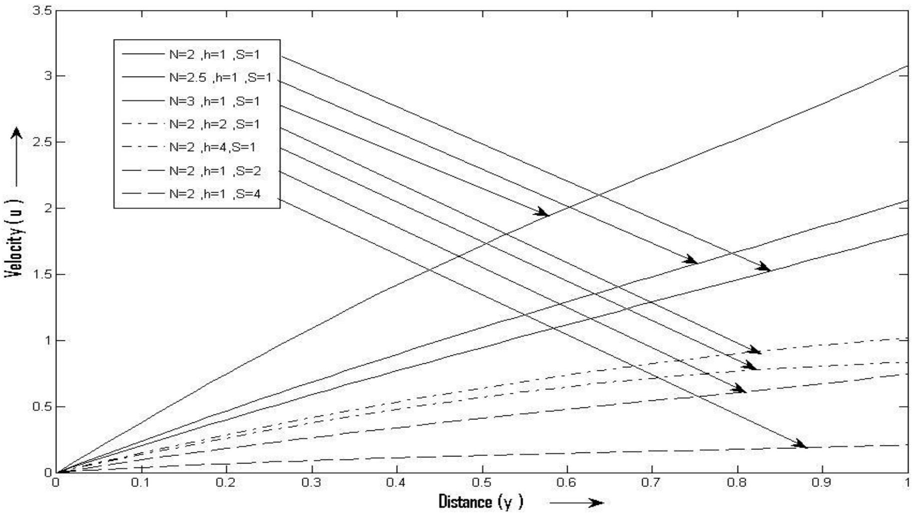

Figure 3: Effect of N, h and S on Velocity Profile when w=1, Re=0.71,Gr=1, t=1, Gm=1, Pe=1, H=1, λ =1 and Sc=0.22

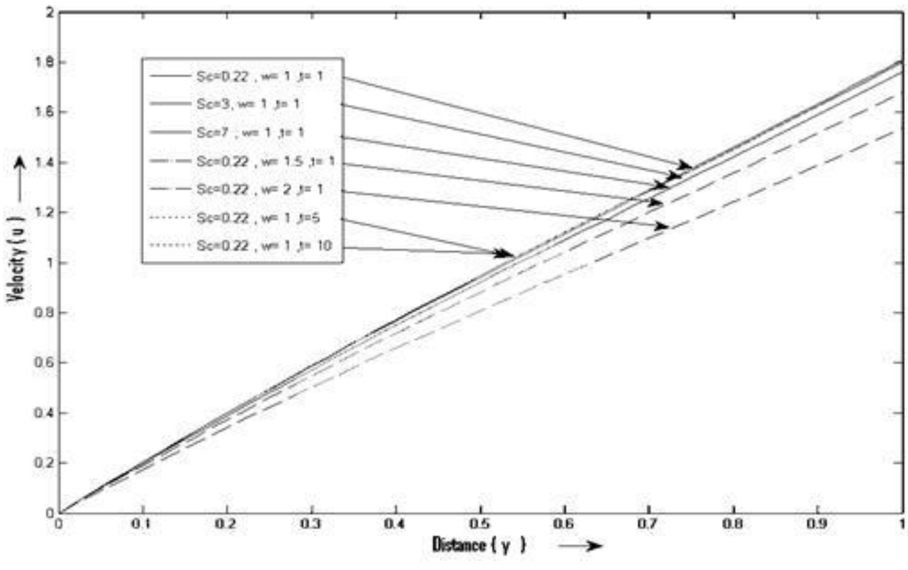

Figure 4: Effect of Sc, w and t on Velocity Profile when H=1, Re=0.71, S=1, h=1, w=1, λ =1, Gr=1, Pe=1. Gm=1 and N=2

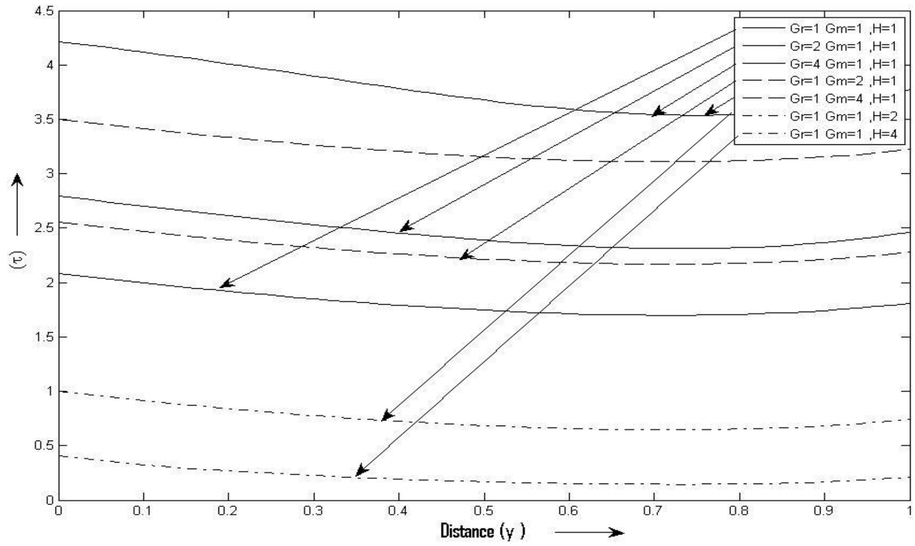

Figure 5: Effect of Gr, Gm and H on Shearing Stress when t=1, Re=0.71, Pe=1, Sc=0.22 h=1, w=1 , S=1, λ =1 and N=2

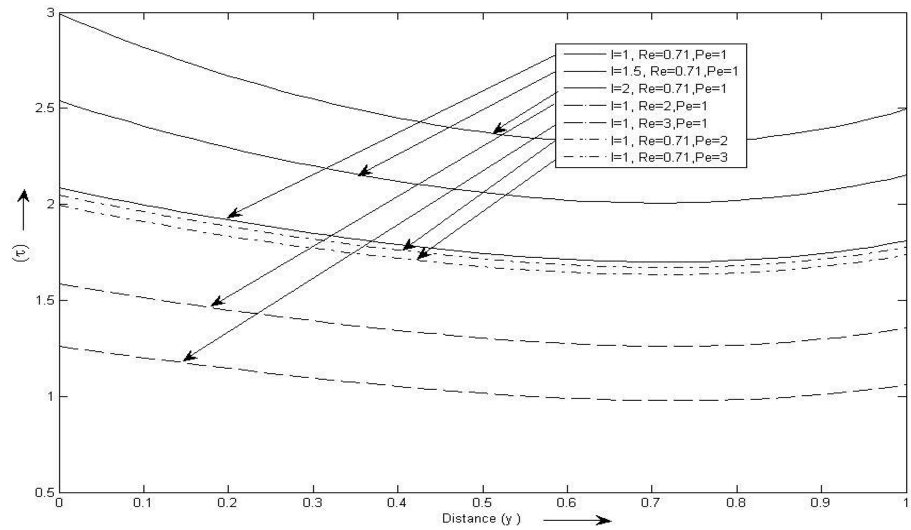

Figure 6: Effect of λ (l),Re and Pe on Shearing Stress when t=1, Gm=1, Gr=1, Sc=0.22 h=1, w=1, S=1H =1 and N=2

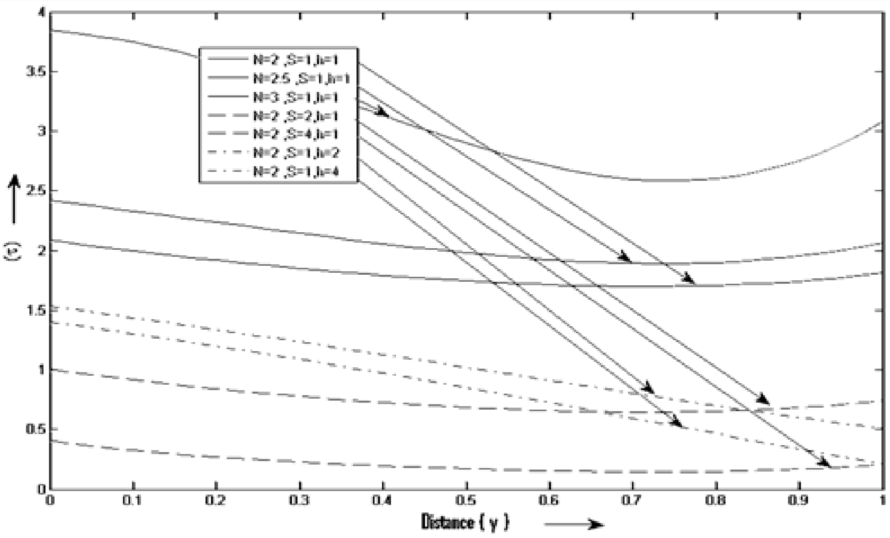

Figure 7: Effect of N, S and h on Shearing Stress when t=1, Gm=1, Gr=1, Re=0.71 Sc=0.22, Pe=1, H =1 and w=1

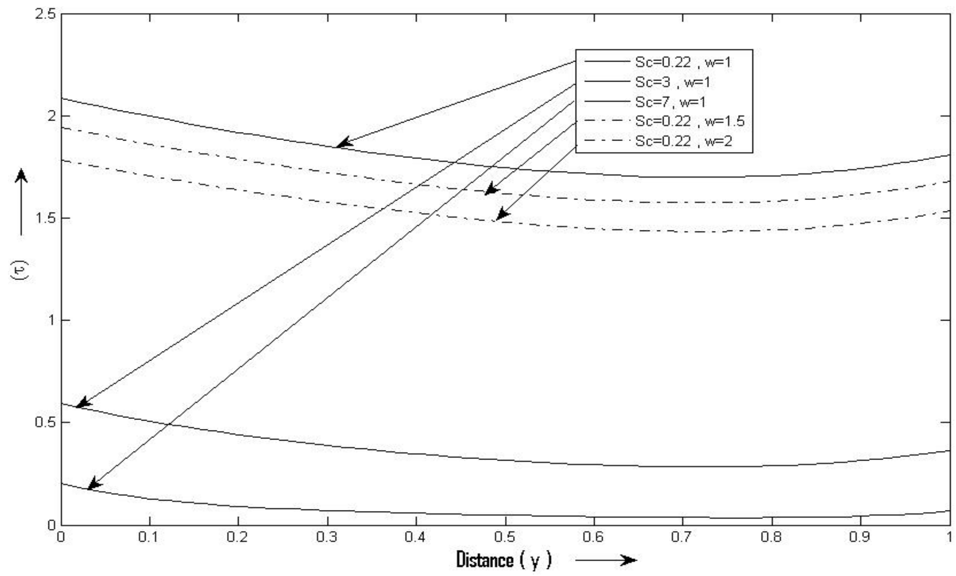

Figure 8: Effect of Sc and w on Shearing Stress when
t=1, Gm=1, Gr=1, Re=0.71 Sc=0.22, Pe=1, H =1 and w=1

CONCLUSIONS

This paper deals with the study of effect of slip condition on unsteady flow of an electrically conducting fluid trough a channel filled with saturated porous medium in the presence of transverse magnetic field, radiative heat transfer and mass transfer. Results are presented graphically to illustrate the variation of velocity, shearing stress, with various parameters. In this study, the following conclusions are set out:

- The velocity increases for the increasing of Gr, Gm,N and λ, also decreases for the increasing of Pe & Re ,Sc ,H,S and h from lower wall to upper wall.
- The shearing stress of both plates increases with the increase of Gr, Gm,N and λ, also decreases for the increasing of Pe & Re ,Sc ,H ,S and h. The shearing stress at upper wall is less than lower wall.

REFERENCES

1. S.Goldstein, moden developments in fluid dynamics ,vol-2 ,p-676,1965.
2. A. C. L. Cogley, W. G. Vinvent, E. S. Giles, American Institue of Aeronautics and Astronautics , 551.1968.
3. S.P.Mishra and T.C. Panda , Oscillatory free convection flow of second order fluid from vertical plate,Proc.indian Acad .Sci.Sect. A,84,20-34 ,1976.
4. E.Ruckenstein,P .Rahora. On the no slip boundary conditions of hydrodynamics,j.colloid interface sci.96,448,1983.
5. V.M.Soundalgekar ,R.M.Lahurikar ,S.G..Pohanerkar andN.S. Birajdar , The effect of mass transfer in the flow past an infinite vertical oscillating plate in presence of constant heat flux,Thermo physics and Aero Mechanics 1,119-124,1994.
6. K.watanche.Yanuar,H.Mizunuma.sip of Newtonian fluids at solid boundary,JSME int.J.Sec. B41, 525,1998.
7. K.watanche.Yanuar,H.Udagawa. Drag reduction of Newtonina fluid in a circular pipe with a highly water-repellant wall,J.Fluid. Mech. 381,225, 1999.

8. W,Marques Jr.G.M.Kremer,F.M.Shapiro,Coutte flow with slip and jump boundary conditions, continumm Mech Thermodynam,12, 379-386,2000.

9. C.Derek,D.C.Tretheway, C.D.Mcinhart.apparent fluid slip athydrophibic microchannelwalls, PhysFluids 14, L19-L12,2002.

10. O. D. Makinde and P. Y. Mhone, Heat transfer to MDH oscillatory flow in a channel filled with porous medium , Rom. Journ. Phys., Vol. 50, Nos. 9–10, P. 931–938, Bucharest, 2005

11. O.D.Makinde,E.Osalusi, MHD steadyflow in achannel with slip at the permeable boundaries. Rom. J.Phys 31(3-4),319-328,2006.

12. N.Senapati, R.K.Dhal.B.Jena, mass and heat transfer effect on mhd oscillotary flow in a channel filled with porous medium Orissa journal of physics,vol-17(1), 2010.